The Arithmetic of Equations

Objectives: Interpret balanced chemical equations in terms of interacting moles, representative particles, masses, and gas volume at STP

- The Story So Far
  - We have learned chemical symbols, charges, how to put together a chemical formula, how to write a chemical reaction and lastly how to balance a chemical reaction
  - Now we enter the world of Stoichiometry – determining how much of an element or compound is needed to make a certain amount of product, or visa versa
- Stoichiometry
  - The calculations of quantities in chemical reactions.
  - We use moles, atoms, molecules, representative particles, grams and liters
  - Take for example the creation of ammonia;
    - \[ N_2(g) + 3H_2(g) \rightarrow 2NH_3(g) \]
    - So what can we get from this balanced equation
    - **Particles** – We can see that 1 molecule of nitrogen reacts with 3 molecules of hydrogen to form 2 molecules of ammonia – based on this there is a 1:3:2 ratio. Since dealing with just a few molecules is impossible, we could scale it up to be 1 Avogadro’s number of nitrogen molecules reacting with 3 Avogadro’s number of hydrogen molecules to form 2 Avogadro’s number molecules of ammonia.
    - **Moles** – Since we know that 1 mole is equal to 1 Avogadro’s number, we can see that it takes 1 mole of nitrogen and 3 moles of hydrogen to form 2 moles of ammonia – a ratio of 1:3:2
    - So we see that in a balanced equation, the coefficients represent the number of moles of each element/compound in the reactant and product – this is very important information. Using this information you can calculate the amount of reactants and products.
    - For the reaction forming NH₃, does the number of moles of reactant equal the number of moles of product? NO
    - **Mass** – remember the law of conservation of mass? It still applies here as well. The sum of the mass of all the reactants must be equal to the mass of the product.
      - So in our example of forming ammonia, 1 mole of nitrogen has a mass of 28.0 g, and 3 moles of hydrogen has a mass of
of 6.0 g for a total of 34.0 g of product. Since ammonia is NH₃, one mole would have a mass of 17.0g, therefore the two moles produced would have a mass of 34.0g, which does equal the mass of the reactants. Mass is indeed conserved

- Volume – As we recently learned, it is easier to express gases in terms of volume. We used STP to determine a common temperature and pressure for measuring volumes. As well we learned that 1 mole of a gas is 22.4 liters.
  - So in the production of ammonia, we see that 1 mole of nitrogen gas (22.4 l) reacts with 3 moles of hydrogen gas (67.2 l) to form 2 moles of ammonia (44.8 l) gas

- The below table shows how mass and atoms are conserved in every reaction, but that molecules, formula units, moles and volumes of gases will not necessarily be conserved

<table>
<thead>
<tr>
<th>N₂(g)</th>
<th>+</th>
<th>3H₂(g)</th>
<th>→</th>
<th>2NH₃(g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 atoms N</td>
<td>+</td>
<td>6 atoms H</td>
<td>→</td>
<td>2 atoms N and 6 atoms H</td>
</tr>
<tr>
<td>1 molecule N₂</td>
<td>+</td>
<td>3 molecules H₂</td>
<td>→</td>
<td>2 molecules NH₃</td>
</tr>
<tr>
<td>10 molecules N₂</td>
<td>+</td>
<td>30 molecules H₂</td>
<td>→</td>
<td>20 molecules NH₃</td>
</tr>
<tr>
<td>1 x (6.02 x 10²³)</td>
<td>+</td>
<td>3 x (6.02 x 10²³)</td>
<td>→</td>
<td>2 x (6.02 x 10²³)</td>
</tr>
<tr>
<td>1 mol N₂</td>
<td>+</td>
<td>3 mol H₂</td>
<td>→</td>
<td>3 mol NH₃</td>
</tr>
<tr>
<td>28.0 g N₂</td>
<td>+</td>
<td>6.0 g H₂</td>
<td>→</td>
<td>34.0 g NH₃</td>
</tr>
</tbody>
</table>

Assume STP

| 22.4 L N₂ | + | 22.4 L H₂ | → | 22.4 L NH₃ |
| 22.4 L N₂ | + | 67.2 L H₂ | → | 44.8 L NH₃ |

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Chemistry Lesson #8 - Stoichiometry
Sample Problems

Hydrogen sulfide, a foul smelling gas (the smell of rotten eggs) is often found around volcano vents. From the balanced equation below, give the interaction of the three relative quantities:

a. number of representative particles
b. number of moles
c. masses of reactants and products

\[ 2H_2S(g) + 3O_2(g) \rightarrow 2SO_2(g) + 2H_2O(g) \]

a. 2 molecules of \( H_2S \) react with 3 molecules of \( O_2 \) to produce 2 molecules of \( SO_2 \) and 2 molecules of \( H_2O \)
b. 2 moles of \( H_2S \) react with 3 moles of \( O_2 \) to produce 2 moles of \( SO_2 \) and 2 molecules of \( H_2O \)
c. multiply the number of moles times the molar mass for each reactant and product

\[
\left(2 \text{ mol} \times \frac{34.1 \text{ g}}{\text{mol}}\right) + \left(3 \text{ mol} \times \frac{32.0 \text{ g}}{\text{mol}}\right) \rightarrow \left(2 \text{ mol} \times \frac{64.1 \text{ g}}{\text{mol}}\right) + \left(2 \text{ mol} \times \frac{18.0 \text{ g}}{\text{mol}}\right)
\]

\[
68.2 \text{ g \( H_2S \)} + 96.0 \text{ g \( O_2 \)} \rightarrow 128.2 \text{ g \( SO_2 \)} + 36.0 \text{ g \( H_2O \)}
\]

Balance the equation for the combustion of acetylene and interpret the equation in terms of relative numbers of moles, volumes of gas at STP, and masses of reactants and products.

\[ 2C_2H_2(g) + 5O_2(g) \rightarrow 4CO_2(g) + 2H_2O(l) \]

\[
\begin{align*}
&2 \text{ mol \( C_2H_2 \)} + 5 \text{ mol \( O_2 \)} \rightarrow 4 \text{ mol \( CO_2 \)} + 2 \text{ mol \( H_2O \)} \\
&44.8 \text{ L \( C_2H_2 \)} + 112.0 \text{ L \( O_2 \)} \rightarrow 89.6 \text{ L \( CO_2 \)} + 44.8 \text{ L \( H_2O \)} \\
&\left(2 \text{ mol} \times \frac{26.0 \text{ g}}{\text{mol}}\right) + \left(5 \text{ mol} \times \frac{32.0 \text{ g}}{\text{mol}}\right) \rightarrow \left(4 \text{ mol} \times \frac{44.0 \text{ g}}{\text{mol}}\right) + \left(2 \text{ mol} \times \frac{18.0 \text{ g}}{\text{mol}}\right) \\
&52.0 \text{ g \( C_2H_2 \)} + 160.0 \text{ g \( O_2 \)} \rightarrow 176.0 \text{ g \( CO_2 \)} + 36.0 \text{ g \( H_2O \)} \\
&212.0 \text{ g products}
\end{align*}
\]

Objectives: Construct mole ratios from balanced chemical equations and apply these ratios in mole-mole stoichiometric calculations; Calculate stoichiometric quantities from balanced chemical equations using units of moles, mass, representative particles and volumes of gases at STP.

- Mole-Mole Calculations
  - With a balanced equation, we know the relationship of how many moles of reactants are needed to make so many moles of product. So if we know the number of moles of one substance, we can determine the number of moles of all other substances in the reaction
  - Look at the production of ammonia again

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Chemistry Lesson #8 - Stoichiometry
- \( N_2(g) + 3H_2(g) \rightarrow 2NH_3(g) \)

- As noted earlier, the most important interpretation of this equation is that 1 mol of \( N_2 \) reacts with 3 moles of \( H_2 \) to form 2 moles of \( NH_3 \) – it is with this ratio knowledge that we can relate the number of moles of reactant to product.

- The coefficients from the balanced equation are used to write conversions called **mole ratios**.

- Mole ratios are used to calculate the number of moles of product from a given number of reactants, or to calculate the number of moles of reactant from a given number of moles of product.

- For the production of ammonia we have a ratio of \( 1N_2:3O_2:2NH_3 \), so we can set up the following ratios:
  - \( \frac{1\text{mol } N_2}{3\text{mol } H_2} \cdot \frac{2\text{mol } NH_3}{2\text{mol } NH_3} \)
  - Or the reverse of these as:
    - \( \frac{3\text{mol } H_2}{1\text{mol } N_2} \cdot \frac{2\text{mol } NH_3}{2\text{mol } NH_3} \)

- So how do we use Mole-Mole Ratios?

**Sample Problems**

How many moles of ammonia are produced when 0.60 mol of nitrogen reacts with hydrogen?

We set up a conversion using one of the mole ratios where we use 0.60 mol \( N_2 \) where the \( N_2 \) will cancel out and leave us with \( NH_3 \) –

\[
0.60\text{mol } N_2 \times \frac{2\text{mol } NH_3}{1\text{mol } N_2} = 1.2\text{mol } NH_3
\]

Given the equation for the formation of aluminum oxide:

\[
4Al(s) + 3O_2(g) \rightarrow 2Al_2O_3(s)
\]

**a.** Write the 6 mole ratios that can be derived

**b.** How many moles of aluminum are needed to form 3.7 mol \( Al_2O_3 \)?

**a.**

\[
\frac{4\text{mol } Al}{3\text{mol } O_2} \cdot \frac{4\text{mol } Al}{2\text{mol } Al_2O_3} \cdot \frac{3\text{mol } O_2}{2\text{mol } Al_2O_3} = \frac{2\text{mol } Al}{4\text{mol } Al} \cdot \frac{2\text{mol } Al_2O_3}{3\text{mol } O_2}
\]

**b.**

\[
3.7\text{mol } Al_2O_3 \times \frac{4\text{mol } Al}{2\text{mol } Al_2O_3} = 7.4\text{mol } Al
\]

According to the above equation:

**a.** How many moles of oxygen are required to react completely with 14.8 mol \( Al \)?

**b.** How many moles of \( Al_2O_3 \) are formed when 0.78 mol \( O_2 \) reacts with aluminum?
a. \[14.8\text{mol Al} \times \frac{3\text{mol } O_2}{4\text{mol Al}} = 11.1\text{mol } O_2\]
b. \[0.78\text{mol } O_2 \times \frac{2\text{mol } Al_2O_3}{3\text{mol } O_2} = 0.52\text{mol } Al_2O_3\]

- Mass-Mass Calculations
  - Scales tend to read in mass, not moles, since that would involve counting atoms/molecules – which we cannot do! So we need to be able to do the same mole-mole calculations with grams, something we can measure
  - The method is one where we convert the mass to moles using molar mass, use the mole ratio, and then convert back to mass using molar mass

Sample Problem
Calculate the number of grams of NH₃ produced by the reaction with 5.40 g of hydrogen with an excess of nitrogen.

\[N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)\] balanced equation

\[5.40\text{g } H_2 \times \frac{1\text{mol } H_2}{2.0\text{g } H_2} = 2.7\text{mol } H_2\text{ convert g } H\text{ to mole } H\]

\[2.7\text{mol } H_2 \times \frac{2\text{mol } NH_3}{3\text{mol } H_2} = 1.8\text{mol } NH_3\text{ find mole of } NH_3\]

\[1.8\text{mol } NH_3 \times \frac{17.0\text{g } NH_3}{1\text{mol } NH_3} = 30.6\text{NH}_3\text{ convert mol } NH_3\text{ to g } NH_3\]

Acetylene gas (C₂H₂) is produced by adding water to calcium carbonate (CaC₂). How many grams of acetylene are produced by adding water to 5.00 g of CaC₂.

\[CaC_2(s) + H_2O(l) \rightarrow C_2H_2(g) + Ca(OH)_2(aq)\]
\[CaC_2(s) + 2H_2O(l) \rightarrow C_2H_2(g) + Ca(OH)_2(aq)\] balanced

\[5.00\text{g } CaC_2 \times \frac{1\text{mol } CaC_2}{64.1\text{g } CaC_2} = 0.078\text{mol } CaC_2\]

\[0.078\text{mol } CaC_2 \times \frac{1\text{mol } C_2H_2}{1\text{mol } CaC_2} = 0.078\text{mol } C_2H_2\]

\[0.078\text{mol } C_2H_2 \times \frac{26.0\text{g } C_2H_2}{1\text{mol } C_2H_2} = 2.03\text{g } C_2H_2\]

Using the same equation, how many moles of CaC₂ are needed to react completely with 49.0g H₂O?
Now that we know how to do mass-mass problems, could we do volume-volume, volume-mass, mass-representative problems? The answer is yes! It is just a matter of conversion.

- Representative particles to moles
- Mass to moles
- Volume to moles
- Returning to moles by
- Moles to representative particles
- Moles to mass
- Moles to volume

Sample Problems

How many molecules of oxygen are produced when a sample of 29.2 g of water is decomposed by electrolysis according to this balanced equation?

\[ 2H_2O(l) \rightarrow 2H_2(g) + O_2(g) \]

\[
29.2\text{g} \ H_2O \times \frac{1\text{mol} \ H_2O}{18.0\text{g} \ H_2O} = 1.62 \text{mol} \ H_2O
\]

\[
1.62 \text{mol} \ H_2O \times \frac{1\text{mol} \ O_2}{2\text{mol} \ H_2O} = .810 \text{mol} \ O_2
\]

\[
.810 \text{mol} \ O_2 \times \frac{6.02 \times 10^{23} \text{molecules} \ O_2}{1 \text{mol} \ O_2} = 4.88 \times 10^{23} \text{molecules} \ O_2
\]

How many molecules of oxygen are produced by the decomposition of 6.54g of potassium chlorate (KClO₃)?

\[ KClO_3 \rightarrow KCl + O_2 \]

\[ 2KClO_3 \rightarrow 2KCl + 3O_2 \]

\[
6.54 \text{KClO}_3 \times \frac{1\text{mol} \ KClO_3}{122.6\text{g} \ KClO_3} = .0533 \text{mol} \ KClO_3
\]

\[
.0533 \text{mol} \ KClO_3 \times \frac{3\text{mol} \ O_2}{2\text{mol} \ KClO_3} = .0800 \text{mol} \ O_2
\]

\[
.0800 \text{mol} \ O_2 \times \frac{6.02 \times 10^{23} \text{molecules} \ O_2}{1 \text{mol} \ O_2} = 4.82 \times 10^{22} \text{molecules} \ O_2
\]

In making nitric acid, the last reaction is nitrogen oxide and water. How many grams of nitrogen must react with water to produce 5.00 x 10^{22} molecules of nitrogen monoxide?
\begin{align*}
3\text{NO}_2(g) + H_2O(l) &\to 2\text{HNO}_3(aq) + NO(g) \\
5.00 \times 10^{22} \text{ molecules NO} \times \frac{1 \text{ mol NO}}{6.02 \times 10^{23} \text{ molecules NO}} &= 8.31 \times 10^{-2} \text{ mol NO} \\
8.31 \times 10^{-2} \text{ mol NO} \times \frac{3 \text{ mol NO}_2}{1 \text{ mol NO}} &= 0.249 \text{ mol NO}_2 \\
0.249 \text{ mol NO}_2 \times \frac{46.0 \text{ g NO}_2}{1 \text{ mol NO}_2} &= 11.5 \text{ g NO}_2
\end{align*}

Assuming STP, how many liters of oxygen are needed to produce 19.8 L SO₃ according to this balanced equation?

\[
2\text{SO}_2(g) + O_2(g) \to 2\text{SO}_3(g)
\]

\[
19.8 \text{ L SO}_3 \times \frac{1 \text{ mol SO}_3}{22.4 \text{ L SO}_3} = 0.884 \text{ mol SO}_3
\]

\[
0.884 \text{ mol SO}_3 \times \frac{1 \text{ mol O}_2}{2 \text{ mol SO}_3} = 0.442 \text{ mol O}_2
\]

\[
0.442 \text{ mol O}_2 \times \frac{22.4 \text{ L O}_2}{1 \text{ mol O}_2} = 9.90 \text{ L O}_2
\]

Using the equation for the combustion of carbon dioxide, how many liters of oxygen are required to completely burn 3.86L of carbon monoxide?

\[
2\text{CO} + O_2 \to 2\text{CO}_2
\]

\[
3.86 \text{ L CO} \times \frac{1 \text{ mol CO}}{22.4 \text{ L CO}} = 0.172 \text{ mol CO}
\]

\[
0.172 \text{ mol CO} \times \frac{1 \text{ mol O}_2}{2 \text{ mol CO}} = 0.0860 \text{ mol O}_2
\]

\[
0.0860 \text{ mol O}_2 \times \frac{22.4 \text{ L O}_2}{1 \text{ mol O}_2} = 1.93 \text{ L O}_2
\]

Limiting Reagent and Percent Yield

Objectives: Identify and use the limiting reagent in a reaction to calculate the maximum amount of product(s) produced and the amount of excess reagent; Calculate the theoretical yield, actual yield, or percent yield given appropriate information.

- Limiting Reagent
  - In working on mole-mole and mass-mass problems, we have made statements similar to “When 6.4 g of N₂ react with an excess of H₂, how much NH₃ is produced?” The key to this statement is “with an excess of H₂”. The answer would be

\[
6.4 \text{ g N}_2 \times \frac{1 \text{ mole N}_2}{28.0 \text{ g N}_2} \times \frac{2 \text{ mol NH}_3}{1 \text{ mol N}_2} \times \frac{17.0 \text{ g NH}_3}{1 \text{ mol NH}_3} = 7.8 \text{ g NH}_3
\]
Suppose, instead of an excess of H\textsubscript{2}, there was only 1.0g available – then how much NH\textsubscript{3} could we have produced? Would 1.0 g H\textsubscript{2} be enough to react completely with 6.4g N\textsubscript{2}?

To answer this problem, we choose one of the reactants and solve for the amount of the other that would be needed for the given amount.

\[
\begin{align*}
6.4\,\text{g}\,\text{N}_2 & \times \frac{1\text{mole}\,\text{N}_2}{28.0\,\text{g}\,\text{N}_2} \times \frac{3\text{mol}\,\text{H}_2}{1\text{mole}\,\text{N}_2} \times \frac{2.0\,\text{g}\,\text{H}_2}{1\text{mole}\,\text{H}_2} = 1.4\,\text{g}\,\text{H}_2 \\
\end{align*}
\]

So we see that to react with 6.4g of N\textsubscript{2} we would need 1.4g H\textsubscript{2} – and we only have 1.0g H\textsubscript{2}. This means that H\textsubscript{2} is the \textbf{limiting reagent}.

On the other hand, since H\textsubscript{2} is the limiting reagent, since we have more than enough N\textsubscript{2} for 1.0g H\textsubscript{2}, then N\textsubscript{2} is the \textbf{excess reagent}.

**Sample Problem**

Sodium chloride can be prepared by the reaction of sodium metal with chlorine gas. Suppose that 6.70 mol of Na reacts with 3.20 mol Cl\textsubscript{2}.

\begin{enumerate}
  \item What is the limiting reagent?
  \item How many moles of NaCl are produced?
\end{enumerate}

\[
\begin{align*}
2\text{Na}(s) + \text{Cl}_2(g) & \rightarrow 2\text{NaCl}(s) \\
6.70\,\text{mol}\,\text{Na} & \times \frac{1\text{mol}\,\text{Cl}_2}{2\text{mol}\,\text{Na}} = 3.35\,\text{mol}\,\text{Cl}_2, \text{Cl}_2 \text{ is the limiting reagent} \\
3.20\,\text{mol}\,\text{Cl}_2 & \times \frac{2\text{mol}\,\text{NaCl}}{1\text{mol}\,\text{Cl}_2} = 6.40\,\text{mol}\,\text{NaCl}
\end{align*}
\]

- Calculating the Percent Yield

When chemical reactions take place, they are almost never 100% complete. A reaction may not go to 100% due to not all the reactants becoming involved, impurities in the reactants, loss of product due to filtering, or just not getting it all out of the vessel.

The predicted amount of product – which we have been doing with our Stoichiometry problems has been for 100% yield, or the \textbf{theoretical yield}.

The \textbf{actual yield} is how much product can be collected – and measured.

The \textbf{percent yield} is a ratio given by:

\[
\text{Percent Yield} = \frac{\text{Actual Yield}}{\text{Theoretical Yield}} \times 100\%
\]

**Sample Problem**

Calcium carbonate is decomposed by heating, as shown in the following equation:

\[
\text{CaCO}_3(s) \xrightarrow{\Delta} \text{CaO}(s) + \text{CO}_2(g)
\]

\begin{enumerate}
  \item What is the theoretical yield of this reaction if 24.8g CaCO\textsubscript{3} is heated to five 13.1g CaO?
  \item What is the percent yield?
\end{enumerate}

\[
\begin{align*}
24.8\,\text{g}\,\text{CaCO}_3 & \times \frac{1\text{mol}\,\text{CaCO}_3}{100.1\,\text{g}\,\text{CaCO}_3} \times \frac{1\text{mol}\,\text{CaO}}{1\text{mol}\,\text{CaCO}_3} \times \frac{56.1\,\text{g}\,\text{CaO}}{1\text{mol}\,\text{CaO}} = 13.9\,\text{g}\,\text{CaO} \\
\text{Percent Yield} & = \frac{13.1\,\text{g}\,\text{CaO}}{13.9\,\text{g}\,\text{CaO}} \times 100\% = 94.2\%
\end{align*}
\]