Physics
Lesson Plan #4
Vector Addition
David V. Fansler
Beddingfield High School

Properties of Vectors

Objectives: How to represent vector quantities graphically and algebraically; Determine the sum of vectors both graphically and algebraically

- Graphical Representation
  - Learned in previous chapter
  - Arrow with head indicate direction
  - Length to indicate magnitude
- Algebraic Representation
  - Displacement + direction in words
    - \( d = 50 \text{ km, southwest} \)
- Resultant Vector
  - Sum of two or more vectors

- Graphical Addition of Vectors
  - Using the above example and graph paper, ruler and protractor
  - Draw 5km E, 4km N and 5km E, draw Resultant
  - Draw 1km N, 8km E, 1km N, 2km E, 2km N, draw resultant
  - Magnitude of resultant is found by measuring the length of the resultant
  - Direction is found using protractor
    - Answer would be 10.77 km 24º north of east

- Special cases
  - If right angles are involved then the Pythagorean theorem can be used – \( R^2 = A^2 + B^2 \)
- Subtracting Vectors
  - Vectors can be multiplied by scalar numbers
    - Changes the magnitude not the direction
    - Unless you multiply by a negative number
    - You can use this to subtract 2 vectors

- Relative Velocities
  - What motions are we under going sitting in the classroom?
    - Earth rotates around it’s axis
    - Earth revolves around the sun
    - The solar system rotates around the center of the galaxy
    - The galaxy is in motion with a local group of galaxies
    - The universe is expanding
  - You can use graphical addition of vectors to solve relative motion problems
  - You are traveling on a school bus that is moving at 8 m/s. You walk toward the front at 3 m/s (relative to the bus)
    - What is your speed relative to the street?
V_{bus} \text{ relative to street} \quad 8 \text{ m/s} \\
V_{you} \text{ relative to bus} \quad 3 \text{ m/s} \\
V_{you} \text{ relative to street} \quad 11 \text{ m/s} \\

- Concept can be used in two dimensions
- Take a sailboat – a side wind will move the boat forward, but also move the boat slightly sideways. To reach a particular destination, the skipper must steer a course that will counteract the sliding.

- Components of Vectors

**Objectives:** Establish a coordinate system in problems involving vector quantities; Use the process of resolution of vectors to find the components of vectors; Determine algebraically the sum of 2 or more vectors by adding the components of the vectors.

- Choosing a Coordinate System
  - Using an x-y coordinate system - there is no right way to set it up (as long as the axis are at right angles to each other)
  - By convention, x increases as it moves to the right from the origin, and y is 90° counterclockwise from the x axis and increases as y moves away from the origin
  - On maps, x points East and y points North
- On problems involving motion through the air, y is the vertical motion and x is the horizontal motion.
- On problems on an incline, +x is set in the direction of motion and y perpendicular to the x axis.

- Components
  - A vector can be broken up into x & y components.

- Here vector \( \mathbf{A} \) is resolved into two component vectors, \( \mathbf{A}_x \) which is parallel to the x axis and \( \mathbf{A}_y \) which is parallel to the y axis.

- \( \mathbf{A} = \mathbf{A}_x + \mathbf{A}_y \)

- Vector resolution is the process of breaking a vector into its components. (\( \mathbf{A}_x \) & \( \mathbf{A}_y \) are called components)

- Algebraic calculations use only the components of vectors – not the vectors themselves.

- Use trigonometry to find the components.
  - \( A_x = A \cos \theta \rightarrow \cos \theta = \frac{\text{adjacent \cdot side}}{\text{hypotenuse}} = \frac{A_x}{A} \)
  - \( A_y = A \sin \theta \rightarrow \sin \theta = \frac{\text{opposite \cdot side}}{\text{hypotenuse}} = \frac{A_y}{A} \)

- When the angle that a vector makes with the x axis is larger than 90º (vector is in 3rd or 4th quadrant) the sign of one or more components is negative.
### Algebraic Addition of Vectors

- Two or more vectors may be added by first resolving each vector into its x & y component.
- Add the x components together, add the y components together.
- For a right triangle, use the Pythagorean theorem:
  - \( R^2 = R_x^2 + R_y^2 \)
  - The angle or direction of the resultant can be found by \( \tan \theta = \frac{R_y}{R_x} \)