Energy and Work

Objectives: Describe the relationship between work and energy; Display an ability to calculate work done by a force; Identify the force that does the work; Differentiate between work and power and correctly calculate power used.

- Energy and Work
  - If you have ever moved, you know what work it is to pick up a box and move it to another location. Sliding it across the floor is not much better due to friction. Working problems in physics also probably seems like hard work – but the meanings of work for moving boxes is not the same as the meaning for working problems.
  - When you describe an object, you might give it’s size, color, weight, and if it can produce a change. The ability of an object to produce a change in itself or it’s environment is called energy.
    - Energy can be in several forms – thermal, chemical, electrical, nuclear, or motion.
    - Let’s look at the energy of motion. If we go back to Newton’s 2nd law, we know that $F = ma$.
    - Using our least favorite velocity equation $v_1^2 = v_0^2 + 2ad$ and rearranging we get $v_1^2 - v_0^2 = 2ad$. And substituting $a = F/m$, we get $v_1^2 - v_0^2 = 2Fd/m$, and finally dividing both sides by 2 we get $\frac{1}{2}m(v_1^2 - v_0^2) = Fd$.
      - The term $\frac{1}{2}mv_1^2$ describes the energy of the system, is called kinetic energy and given the symbol $K$.
      - The right hand side of the equation $Fd$ refers to the environment, a force through a displacement. This means that some agent in the environment changed a property of the system. As we noted earlier, changing the energy of a system is called work and is given the symbol $W$, where $W = Fd$.
      - If we substitute $W$ and $K$ into the original equation, we end up with $K_1 - K_0 = W$, or work is the change in kinetic energy so $W = \Delta K$.
      - $W = \Delta K$ is called the work-energy theorem.
      - The definition of the unit of measure for $W$ is 1kg moved at one m/s has a kinetic energy of 1kg·m$^2$/s$^2$, known as 1 Joule or 1J.
      - An apple weighs about 1 N, so when you lift it a distance of one meter, then you do 1J of work on it.
  - Calculating Work
While the equation for work is \( W = Fd \), this only works for constant forces exerted in the direction of the motion.

Sample Problem
A 105g hockey puck is sliding across the ice. A player exerts a constant 4.5N force over a distance of 0.15m. How much work does the player do on the puck? What is the change in the puck's energy?

Since the force and the direction of motion are the same, all the units would be positive

\[
W = 4.5 \, \text{N} \times 0.15 \, \text{m} = 0.68 \, \text{N} \times \text{m} = 0.68 \, \text{J}
\]

And since \( \Delta K = W \), then change in energy would be 0.68J

- **Constant Force at an Angle**
  - In talking about calculating work, we indicated that the constant force had to be in the direction of motion. So how about the situation where you are pushing a lawnmower? The force you are applying is not in the direction of motion – rather at an angle of about 25° to the motion, so are you doing any work? Of course you are! Back to trigonometry!

  As you can see we need to find the component of force in the x-axis, since that is the direction of motion of the mower. \( F_x = F \cos \Theta \)

  Inserting this into our \( W = Fd \) equation we get \( W = Fd \cos \Theta \)

- **What about other forces on the lawnmower?**
  - Gravity is exerting a force, but it is balanced by the normal force and besides, there is no component of it in the horizontal direction
  - There is friction and it is always opposite the direction of motion, and therefore would have negative sign. Negative work done by a force in an environment reduces the energy of the system – in this case, just makes the person have to push the mower with more force, doing more work!

- **Finding Work when Forces Change**
  - We have been careful to state that “when a constant force is applied . . .”, and we can see why when we look at a graph of a constant force over a distance. The work would simply be the force times the distance – but
what about when the force is not constant? Suppose that we apply a force of 20N to an object over a distance 1.5m, the work done would be simple to calculate – the force times the distance. But what if we started out with a force of 0N and increased it evenly to 20N while we pushed the object 1.5m?

![Force vs. Distance Graphs](image)

**Energy, Work and Simple Machines - Figure 2**

- For the constant force case we see that the force times the distance is the area under the curve. We can use the same concept for a force that is changing. When the force is plotted against the distance, if we calculate the area under the graph we can get the work.
  - In this case, we are looking at a triangle, which the area is ½ lh, or \( W = \frac{1}{2} Fd \). So \( W = \frac{1}{2} (20N)(1.5m) = 15N \cdot m = 15J \)

- **Power**
  - Up until now, we have not discussed how long it has taken the work to take place. Does it matter? If you walk up a flight of stairs, you are doing work, but what if you run up the same stairs? Do you do the same amount of work? Yes you do, but the **power** is different. Power is a measure of how much work is done in a period of time – the rate of work. The longer it takes to do work, the less power. Power is designated by the letter \( P \).

  The equation for power is \( P = \frac{W}{t} \). The unit of measure for power is the watt (W). One watt is 1J of energy transferred in one second. A glass of water weighs about 2N, and if you move it .5 m to your mouth, then you have done 1J of work. If it takes 1 second to move the glass then you are working at the rate of 1W. Since the watt is such a small unit, power is more often expressed in kilowatts (1000W)

**Sample Problem**

An electric motor lifts an elevator 9.00 m in 15.0s by exerting an upward force of 1.20 x 10^4N. What power does the motor produce in watts and kilowatts?

Using the definitions of work (\( W = Fd \)) and power (\( P = \frac{W}{t} \)) we can solve separately or combine the equations to solve at one time (\( P = \frac{Fd}{t} \))

\[
P = \frac{Fd}{t} = \frac{(1.20 \times 10^4N)(9.00m)}{15.0s} = 7200W = 7.2kW
\]

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Physics Lesson #9 – Energy, Work and Simple Machines
Machines

Objectives: Demonstrate knowledge of why simple machines are useful; Communicate an understanding of mechanical advantage in ideal and real machines; Analyze compound machines and describe them in terms of simple machines; Calculate efficiencies for simple and compound machines.

- Simple and Compound Machines
  
  Once upon a time, soft drinks had caps on them that required a special device to open them, called a bottle opener. The bottle opener would grab under the cap and lift the cap off the bottle. The cap could not be taken off the bottle by bare hands alone!

  ![Diagram of a bottle opener](image)

  - This is an example of a simple machine. The force you exert on the end of the opener is the **effort force** – $F_e$. The force that is exerted by the machine is the **resistance force** – $F_r$. The ratio of the resistance force to the effort force is called the mechanical advantage. $MA = \frac{F_r}{F_e}$

  - Many machines have a mechanical advantage greater than one – when $MA$ is greater than one, then the machine increases the amount of force you apply.

  - Some machines (such as a simple pulley) have a MA of 1 – while there is no increase in force, there is a redirection of the force which can be helpful.

  - You can write the mechanical advantage of a machine using the definition of work. Since you exert a force through a distance, the work in ($W_i$) would be $F_e d_e$. The output of a machine is going to be a force through a distance, so work out ($W_o$) would be $F_r d_r$. In an ideal world $W_i = W_o$, or $F_e d_e = F_r d_r$. We can re-write this equation to be $\frac{F_r}{F_e} = \frac{d_e}{d_r}$. We know that mechanical advantage is $MA = \frac{F_r}{F_e}$, and for an ideal machine then $MA = \frac{d_e}{d_r}$, but because this is for an ideal machine then the mechanical advantage is called the **ideal mechanical advantage** and is written $IMA = \frac{d_e}{d_r}$. From this equation, we see that you measure the distances to calculate the IMA, but you have to measure the forces exerted to find the actual $MA$. 

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Physics Lesson #9 – Energy, Work and Simple Machines
- Efficiency
  - Since in the real world there are no ideal machines, there will always be some forces lost (typically to friction). Any forces lost means that not all the energy put into a system will be more than the forces taken out of the system.
  - The efficiency of a machine is defined as a ratio of the output work to the input work - \( \text{efficiency} = \frac{W_o}{W_i} \times 100\% \), or we can re-write this to be
    \[
    \text{efficiency} = \frac{F_r / F_e}{d_e / d_r} \times 100\% \text{ and once more as } \text{efficiency} = \frac{MA}{IMA} \times 100\%
    \]
  - An efficient machine will be close to 100% effective.
- Simple Machines
  - All machines, no matter how complex, are a combination of one or more of the six simple machines
    - Lever
    - Pulley
    - Wheel and axle
- Inclined plane

- Wedge

- Screw
You can see these simple machines in many items, such as a bicycle – what machines do you see in a bicycle?

- Wheel and Axle, for the pedal and rear wheel
- Lever for the handle bars

The IMA of all machines is the ratio of the distances moved. When talking about wheel and axle, you use the distance between where the force is applied and the rotational point.

### Compound Machines

- A compound machine consists of 2 or more simple machines, linked so the resistive force of the first machine becomes the effort force of the second.
  - An example would be a bicycle – the pedal and gear act as wheel and axle. The force you place on the pedal becomes the resistance force on the gear, but through the chain this resistance force becomes the effort force for the gear on the rear wheel and the resistance force is the wheel on the ground.

The mechanical advantage of a compound machine is the product of the mechanical advantages of the simple machines that make it up, such that

\[ MA = MA_{machine1} \times MA_{machine2} \]

- If we take the example of a bicycle, then we have

\[ MA = \frac{F_{on\ chain}}{F_{on\ pedal}} \times \frac{F_{on\ road}}{F_{by\ chain}} = \frac{F_{on\ road}}{F_{on\ pedal}} \]

- For calculating the IMA of each wheel and axle, it is the ratio of the distances moved,
  - So the pedal and gear you would have

\[ IMA = \frac{\text{pedal radius}}{\text{front sprocket radius}} \]
And for the rear wheel you would have

\[ IMA = \frac{\text{rear sprocket radius}}{\text{wheel radius}} \]

Then finding the product we would have

\[ IMA = \frac{\text{pedal radius}}{\text{front sprocket radius}} \times \frac{\text{rear sprocket radius}}{\text{wheel radius}} \]

which we can rearrange to be

\[ IMA = \frac{\text{pedal radius}}{\text{wheel radius}} \times \frac{\text{rear sprocket radius}}{\text{front sprocket radius}} \]

Since both sprockets use the same chain, then we can count the teeth and use that as a measurement and rewrite once again to

\[ IMA = \frac{\text{pedal radius}}{\text{wheel radius}} \times \frac{\text{teeth on rear sprocket}}{\text{teeth on front sprocket}} \]

Shifting gears on your bicycle is a way of adjusting the ratio of sprocket radii to obtain the desired IMA.

You should remember that forces are always taken to be in the direction of motion, so the most force you can place on a pedal of a bicycle is when the pedal arm is horizontal to the ground. When ever force on a bicycle is specified, it should be taken that the pedal is horizontal to the ground.

Example Problem

While examining a rear wheel of a bicycle, you find that it has a radius of 35.6 cm and an attached gear which has a radius of 4.00 cm. When the chain is pulled with a force of 155 N, the wheel rim moves 14.0 cm. The efficiency of this part of the bicycle is 95.0%.

a. What is the IMA of the wheel and gear?
b. What is the MA of the wheel and gear?
c. What force does the scale register?
d. How far was the chain pulled to move the rim that amount?

Using the equation

\[ IMA = \frac{d_e}{d_r} = \frac{4.00 \text{ cm}}{35.6 \text{ cm}} = 0.112 \]
Since we know that \( \text{efficiency} = \frac{MA}{IMA} \times 100\% \), the we can rearrange to find \( MA \),

\[
MA = \frac{\text{efficiency} \cdot IMA}{100\%} = \frac{95.0\% \cdot 0.112}{100\%} = .106
\]

To find what force the scale registers we use the \( MA = \frac{F_r}{F_e} \) equation, where it can be re-written

\[
F_r = MA \cdot F_e = 0.106 \cdot 155 N = 16.4 N
\]

And lastly to find the length of chain pulled, we would go back to \( IMA = \frac{d_e}{d_r} \) and re-write it to

\[
d_e = IMA \cdot d_r = .112 \cdot 14.0 cm = 1.57 cm
\]